

## A Chameleon Primer

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We review some of the properties of chameleon theories. Chameleon fields are gravitationally coupled to matter and evade gravitational tests thanks to two fundamental properties. The first one is the density dependence of the chameleon mass. In most cases, in a dense environment, chameleons are massive enough to induce a short ranged fifth force. In other cases, non-linear effects imply the existence of a thin shell effect shielding compact bodies from each other and leading to an irrelevant fifth force. We also mention how a natural extension of chameleon theories can play a role to solve the PVLAS versus CAST discrepancy.

### 1 Introduction

Fifth force experiments such as the Cassini satellite experiment put stringent bounds on the gravitational coupling of nearly massless scalar particles. Future satellite tests of fifth forces and putative violations of the equivalence principle will even lead to stronger constraints. As no scalar field has ever been observed, these bounds would not be so dramatic if the existence of nearly massless fields was not suggested by the late time acceleration of the universe expansion<sup>1,2</sup>. In fact, these constraints have fundamental consequences for models of dark energy. Indeed, models of dark energy known as quintessence<sup>3,4,5,6</sup> require the existence of a runaway scalar field with a tiny mass now, of order  $H_0 \sim 10^{-43}$  GeV. The range of the interactions mediated by the quintessence scalar field is of order of the Hubble horizon size.

Hence, unless the quintessence field has a very small gravitational interaction with ordinary matter, fifth force experiments are not compatible with a quintessence scenario. For instance, embedding quintessence models in spontaneously broken supergravity proves to be extremely difficult as the gravitational couplings are generically large<sup>7</sup>. Within string theory, the dilaton has been argued to be a quintessence candidate provided the coupling to matter is universal and possesses a minimum playing the role of an attractor where all gravitational problems are evaded<sup>8</sup>. Of course, it would be extremely interesting to confirm this possibility explicitly. String moduli fields are also natural candidates for quintessence. Unfortunately, their gravitational coupling is generically of order one. On the other hand, there exists a well-motivated scalar field with a small gravitational field: the radion measuring the inter-brane distance in Randall-Sundrum scenarios. In this case, the gravitational coupling of the radion to matter on a warped brane is suppressed by the warp factor and becomes very small for a large radion.

Chameleon field theory combine both a quintessence-like behaviour leading to dark energy

at late time and a gravitational coupling to matter which can be large<sup>9,10</sup>. So how come they are not definitely ruled out by fifth force experiments? In fact, it is useful to draw an analogy with photons. In some circumstances, photons do get a mass which alters their properties. This is notoriously the case in superconductors where the Meissner effect (the fact that the magnetic field is expelled from a superconductor) can be seen as the result of the Higgs mechanism with a mass given to the photons<sup>11</sup>. In less extreme situations, like in a crystal, photons are slowed down when interacting with matter. Similar phenomena can occur for scalar fields. Typically, scalar particles have an effective potential obtained as a combination of the bare potential appearing in the Lagrangian and a term proportional to the matter density. This effective potential may have a density dependent minimum. In this case, we call the field a chameleon as its mass depends on the environment.

Chameleon fields are generically more massive in a dense environment. This is enough to evade the gravitational bounds in most cases. Indeed, the range of the chameleon mediated force becomes too small to be detected. Even when this is not the case, chameleon theories may enjoy another non-trivial property: the existence of thin shells. More precisely, the field created by a massive object may be essentially trapped inside the massive body. In this case, the interaction between massive bodies is essentially non-existent. Combining these two effects, one can build satisfying examples of chameleon theories. We will review their main properties here.

Recently, the PVLAS experiment has measured the dichroism of light propagating through a magnetic field<sup>12</sup>. This can be understood by coupling a scalar field to photons. In this case, one can use the environment dependent mass of chameleon fields to generate a large mass for the chameleon in the sun. Therefore, chameleons would not be produced by the Primakov effect and therefore the CAST experiment<sup>13</sup> would not see the photon regeneration by inverse Primakov effect. We will present some of these ideas very briefly.

## 2 Scalar-Tensor Theories

### 2.1 Coupling to matter

Chameleon fields appear in scalar-tensor theories of gravity<sup>14</sup>. We start with a discussion of these theories. We consider theories where a scalar field  $\phi$  couples both to gravity and matter, generating a potential fifth force. The Lagrangian of such scalar-tensor theories reads

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} (R - (\partial\phi)^2 - 2\kappa_4^2 V(\phi)) \quad (1)$$

Matter couples to both gravity and the scalar field according to

$$S_m(\psi, A^2(\phi)g_{\mu\nu}), \quad (2)$$

where  $\psi$  is a matter field and  $A$  is an arbitrary function of  $\phi$ . The Klein-Gordon equation can be written in terms of an effective potential

$$V_{\text{eff}}(\phi) = V(\phi) + \rho_m A(\phi). \quad (3)$$

The effective potential depends on the environment through the matter energy density  $\rho_m$ . We will assume that  $V(\phi)$  is a runaway potential and for the models we consider  $A(\phi)$  increases with  $\phi$ . In that case the potential has a minimum whose location depends on  $\rho_m$ , i.e. on the environment. Such a field has been called a chameleon field.

The field  $\phi$  acts on all types of matter and, in the Einstein frame, there is a new force associated with the scalar field

$$F_\phi = -\kappa_4 m \alpha_\phi \frac{\partial\phi}{\partial x_\mu}, \quad (4)$$

where  $m$  is the mass of the test particle in the Einstein frame and

$$\alpha_\phi = \frac{\partial \ln A}{\partial \kappa_4 \phi} \quad (5)$$

The force  $F_\phi$  cannot be too large, otherwise experiments would have already detected it.

For massless fields  $V(\phi) \equiv 0$  and a point-like matter source, the Klein-Gordon equation becomes

$$\Delta \phi = -\kappa_4 m \alpha_\phi|_{r=0} \delta^{(3)}(r) \quad (6)$$

where  $m = A(\phi)|_{r=0} m_0$  is the Einstein frame mass and  $m_0$  the bare mass of the source. The resulting field  $\phi = -\kappa_4 \alpha_\phi|_{r=0}/4\pi r$  leads to a force between bodies  $F_\phi = 2G_N \alpha_\phi|_{r=r_1} \alpha_\phi|_{r=r_2} m_1 m_2 / r_{12}$  where  $\kappa_4^2 = 8\pi G_N$ . This produces a fifth force where

$$F_\phi = 2\alpha_1 \alpha_2 F_{\text{Newton}} \quad (7)$$

and  $\alpha_1 = \alpha_\phi|_{r=r_1}$ . The Cassini experiments impose that  $\alpha_\phi^2 \leq 5 \cdot 10^{-5}$  for a constant coupling. Hence massless particles (or nearly massless particles with a mass less than  $10^{-3}$  eV) must have a very small coupling to gravity. Chameleon field theories enable to overcome this obstacle.

## 2.2 The radion

A simple and interesting example of non-trivial coupling to gravity is provided by the Randall-Sundrum scenario where matter is confined on 4d hyperplanes embedded in an  $AdS_5$  vacuum<sup>15</sup>. The two boundaries of space-time are called the UV and the IR brane reflecting the fact that the metric is warped. Distances on the IR branes are warped down compared to scales on the UV. Consider now matter on the UV brane of positive tension. The coupling of matter to gravity depends on the radion field  $\phi$  (for a derivation of the following equations and references, see e.g. the review<sup>16</sup>)

$$A(\phi) = \cosh \frac{\kappa_4 \phi}{\sqrt{6}} \quad (8)$$

where the inter-brane distance is

$$d = -l \ln(\tanh \frac{\kappa_4 \phi}{\sqrt{6}}) \quad (9)$$

For small distances compared to the AdS curvature  $l$ , the coupling becomes

$$A(\phi) = \frac{1}{2} e^{\kappa_4 \phi / \sqrt{6}} \quad (10)$$

The gravitational coupling is constant

$$\alpha_\phi = \frac{1}{\sqrt{6}} \quad (11)$$

Of course, this is too big for the Cassini bound. However, in this case and in the large interbrane-distance limit, the chameleon mechanism can be applied to hide the interaction mediated by the radion<sup>17</sup> by introducing a bare potential for the radion field.

## 2.3 Chameleon Cosmology

We concentrate on a particular model where  $A(\phi) = e^{\beta \phi}$  and  $\beta = O(1)$ . We consider the family of potential

$$V = M^4 f\left(\left(\frac{M}{\phi}\right)^n\right) \quad (12)$$

where  $f$  leads to ordinary quintessence with a long time tracking solution. A typical example is provided by  $f(x) = e^x$ . As  $\phi \gg M$  now, the potential is nothing but

$$V = M^4 + \frac{M^{4+n}}{\phi^n} \quad (13)$$

Cosmologically, it mimics a cosmological constant. For gravitational tests, only the Ratra–Peebles part of the potential matters.

This model satisfies the chameleon property of having a  $\rho$ -dependent minimum. As  $\beta = 0(1)$ , the coupling of matter to the chameleon field is large and may be in conflict with experiments. We will study the gravitational aspects in the next section. Here we concentrate on cosmological properties.

In a Friedmann–Robertson–Walker Universe, the (non)-conservation of matter equation reads

$$\dot{\rho} + 3H\rho = \alpha_\phi \dot{\phi} \rho. \quad (14)$$

leading to

$$\rho = A(\phi)\rho_m, \quad \rho_m = \frac{\rho_0}{a^{3(1+w_m)}} \quad (15)$$

while the Klein–Gordon equation can be written in terms of an effective potential

$$V_{\text{eff}}(\phi) = V(\phi) + \rho_m(1 - 3w_m)A(\phi). \quad (16)$$

Let us now go through the different cosmological eras<sup>10</sup>. During inflation the chameleon potential has an effective minimum which is time-independent. Moreover, as the mass of the chameleon field at the minimum is  $m \gg H$ , the field oscillates rapidly and converges to the minimum extremely rapidly, behaving like a dust component. By the end of inflation, the field is stuck at the minimum. As inflation stops and the radiation era starts, the minimum is pushed far away (as it depends only of non-relativistic matter). The field is therefore in an overshooting regime where it becomes kinetically dominated, being far away to the right of the potential. The field overshoots before stopping at  $\phi_{\text{stop}} = \phi_{\text{in}} + \sqrt{6\Omega_\phi^i} m_{\text{Pl}}$  where  $\Omega_\phi^i$  is the initial chameleon fraction density. After stopping the field is in an undershooting position. In that case, the field would remain still until either being caught up by the minimum or the beginning of the matter era. When caught by the minimum the field oscillate and converges to the minimum, which is a tracker solution. This follows from the fact that  $m \gg H$  at the minimum throughout the history of the Universe. The field converges to the minimum faster than  $a^{-3}$  due to the time variation of the mass at the minimum.

In fact if the field is far away from the minimum after overshooting, it is sensitive to short bursts when relativistic particles become non-relativistic

$$\ddot{\phi} + 3H\dot{\phi} = \frac{\beta}{m_{\text{Pl}}} T_\mu^\mu \quad (17)$$

as, during such periods, the trace of the energy momentum tensor  $T_\mu^\mu$  of the decoupling species is temporarily non-vanishing, resulting in a kick<sup>18</sup> of order of a fraction of  $\beta$ . Taking into account all these kicks, the field decreases by about  $\Delta\phi \sim -\beta m_{\text{Pl}}$ . By BBN, either the field is close to the minimum, in which case the electron kick which occurs during BBN does not lead to a large variation of  $\phi$  during BBN, or the field is still far away from the minimum in which case the electron kick leads to large variations of  $\phi$  and therefore of masses

$$\left| \frac{\Delta m}{m} \right| = \beta \left| \frac{\Delta\phi}{m_{\text{Pl}}} \right| \quad (18)$$

the latter case being excluded. As a result, the initial value of  $\phi$  cannot be larger than one and  $\Omega_\phi^i \leq 1/6$ , a weaker bound than in quintessence. Once at the minimum by BBN, the field follows the attractor in the matter era. Once the vacuum energy dominates, the matter density decreases extremely fast. The chameleon field follows the minimum until  $m \sim H$  where it starts lagging behind eventually having the same evolution as a quintessence field with no coupling to matter.

### 3 Gravitational Tests

#### 3.1 The massive chameleon

The effective potential with  $f(x) = e^x$  leads to a stabilisation of the scalar field for

$$\phi = \left( \frac{n\Lambda^{4+n}M}{\rho} \right)^{1/(n+1)}, \quad (19)$$

where  $\rho$  is the matter energy density. The mass at the bottom of the potential is given by

$$m^2 = n(n+1) \frac{\Lambda^{n+4}}{\phi^{n+2}} \quad (20)$$

In the atmosphere, the mass of chameleons is larger than  $10^{-3}$  eV implying no consequence for Galileo's Pisa experiment and similar tests.

#### 3.2 The thin shell

Let us now consider a situation where the gravitational experiments are performed on a body embedded in a surrounding medium. The body could be a small ball of metal in the atmosphere or a planet in the inter-planetary vacuum. The effective potential (16) is not the same inside the body and outside because  $\rho_m$  is different. The effective potential can be approximated by

$$V_{\text{eff}} \simeq \frac{1}{2} m_\phi^2 (\phi - \phi_{\text{min}})^2, \quad (21)$$

As already mentioned the minimum and the mass are different inside and outside the body. We denote by  $\phi_b$  and  $m_b$  the minimum and the mass in the body and by  $\phi_\infty$  and  $m_\infty$  the minimum and the mass of the effective potential outside the body. Then, the Klein-Gordon equation reads

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = \frac{\partial V_{\text{eff}}}{\partial \phi}, \quad (22)$$

where  $r$  is a radial coordinate. Requiring that  $q$  remains bounded inside and outside the body and joining the interior and exterior solutions, one can determine the complete profile which can be expressed as

$$\begin{aligned} \phi_{<}(r) &= \phi_b + \frac{R_b (\phi_\infty - \phi_b) (1 + m_\infty R_b)}{\sinh(m_b R_b) [m_\infty R_b + m_b R_b \coth(m_b R_b)]} \frac{\sinh(m_b r)}{r}, & r \leq R_b, \\ \phi_{>}(r) &= \phi_\infty + R_b (\phi_b - \phi_\infty) \frac{m_b R_b \coth(m_b R_b) - 1}{[m_\infty R_b + m_b R_b \coth(m_b R_b)]} \frac{e^{-m_\infty(r-R_b)}}{r}, & r \geq R_b \end{aligned} \quad (23)$$

Assuming, as it is always the case in practise, that  $m_b \gg m_\infty$ ,  $m_b R_b \gg 1$ , one has

$$\frac{\partial \phi_{>}(r)}{\partial r} \simeq -\frac{R_b}{r^2} (\phi_\infty - \phi_b), \quad (24)$$

from which we deduce that the acceleration felt by a test particle is given by

$$a = \frac{G_N m_b}{r^2} \left[ 1 + \frac{\alpha_\phi (\phi_\infty - \phi_b)}{\Phi_N} \right], \quad (25)$$

where  $\Phi_N = G_N m_b / R_b$  is the Newtonian potential at the surface of the body. Therefore, the theory is compatible with gravity tests if

$$\frac{\alpha_\phi (\phi_\infty - \phi_b)}{\Phi_N} \ll 1. \quad (26)$$

Large compact bodies have a thin shell implying that no distortion of solar system planetary orbits are predicted. Lunar ranging experiments are not affected either.

### 3.3 Chameleon in a cavity

Gravitational experiments on earth and future satellite experiments involve vacuum chambers which can be modelled out as spherical cavities of radius  $R$ . Solving the chameleon equations in this situation, following the same method as in the previous subsection, we find that the mass of the chameleon field inside the cavity is determined by the resonance equation

$$\frac{\sinh m_0 R}{m_0 R} = n + 2 \quad (27)$$

Having determined  $m_0$ , one can deduce the value of the field  $\phi_0$  inside the cavity. Notice that for most values of  $n$  we have

$$m_0 R = O(1) \quad (28)$$

When  $\beta = O(1)$ , the mass of the chameleon in gravitational experiments on earth is of order  $1/R$  and is too large to evade gravitational tests (the range is given by  $R \sim 1$  m). Fortunately, typical test bodies on earth have a thin shell implying no deviation from Newton's law for Eotvos or Eotwash experiments<sup>19</sup>. Future satellite experiments are such that test bodies do not have a thin shell. Hence large deviations from Newton's law are predicted. When  $\beta \gg 1$ , test bodies have a thin shell and satellite experiments would not see any deviation. (For a discussion of the case  $\beta \gg 1$ , see reference<sup>20</sup>).

## 4 PVLAS vs CAST

Recently, the coupling of a scalar field to photons have been invoked in order to explain the PVLAS results on dichroism<sup>12</sup>. The scalar field is required to have a mass of order  $10^{-12}$  GeV and a coupling strength suppressed by a scale of order  $M = 10^6$  GeV. The coupling to photons is given by

$$-\frac{1}{4} \int d^4x e^{\phi/M} F_{\mu\nu} F^{\mu\nu} \quad (29)$$

The results of the PVLAS collaboration are in conflict with astrophysical bounds such as CAST<sup>13</sup>, which for the same mass for the scalar field, require much smaller couplings ( $M > 10^{10}$  GeV).

The chameleon mechanism can help in explaining the PVLAS results and, at the same time, be in agreement with astrophysical bounds<sup>21</sup>. Our model is of the scalar-tensor type

$$\begin{aligned} S = & \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa_4^2} R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{e^{\phi/M}}{4} F^2 \right) \\ & + S_m(e^{\phi/M} g_{\mu\nu}, \psi_m) \end{aligned} \quad (30)$$

where  $S_m$  is the matter action and the fields  $\psi_m$  are the matter fields. The effective gravitational coupling is given by

$$\beta = \frac{m_{\text{Pl}}}{M}, \quad (31)$$

and therefore very large ( $\beta = 10^{13}$ ) when assuming the results from the PLVAS experiment ( $M = 10^6$  GeV). To prevent large deviations from Newton's law one must envisage non-linear effects shielding massive bodies from the scalar field. One natural possibility is that the scalar field  $\phi$  coupled to photons has a runaway (quintessence)-potential leading to the chameleon effect. For exponential couplings, this is realised when

$$V(\phi) = \Lambda^4 \exp(\Lambda^n/\phi^n) \approx \Lambda^4 + \frac{\Lambda^{4+n}}{\phi^n} \quad (32)$$

In the presence of matter, the dynamics of the scalar field is determined by an effective potential

$$V_{\text{eff}}(\phi) = \Lambda^4 \exp(\Lambda^n/\phi^n) + e^{\phi/M} \left( \rho + \frac{\mathbf{B}^2}{2} \right) \quad (33)$$

where  $\rho$  is the energy density of non-relativistic matter.

As already mentioned, the PVLAS experiment is in conflict with the CAST experiment on the detection of scalar particles emanating from the sun, as it requires  $M \geq 10^{10}$  GeV. However, this bound does not apply when the mass of the scalar field in the sun exceeds  $10^{-5}$  GeV. Let us evaluate the mass of the chameleon field inside the sun. Furthermore, from the effective potential one obtains

$$m_{\text{sun}} = m_{\text{lab}} \left( \frac{\rho_{\text{sun}}}{\rho_{\text{lab}}} \right)^{(n+2)/2(n+1)}. \quad (34)$$

Now  $\rho_{\text{sun}}/\rho_{\text{lab}} \approx 10^{14}$  and, with  $n = 0(1)$ , one finds

$$m_{\text{sun}} \sim 10^{-2} \text{ GeV} \gg 10^{-5} \text{ GeV} \quad (35)$$

implying no production of chameleons in the sun. Hence, the CAST experiment is in agreement with the chameleon model due to the fact that the chameleon field is very massive in the sun.

## 5 Conclusion

We have given an brief overview of chameleon field theories. They provide exciting new mechanisms for both gravitation and cosmology.

A scalar field coupled to matter can be problematic, since it mediates a new force. But if the field self-interacts in a non-linear way, as it is the case in chameleon field theories, the effect of the field can be hidden from current experiments. As we pointed out, future experiments will be able to search for such chameleon fields. We have speculated that the PVLAS anomaly finds a natural interpretation within these theories.

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